

NASA TECHNICAL  
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N71-26979

NASA TM X-2297

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ANALYTIC STUDY OF AN ELECTRIC  
STEPPING MOTOR DRIVE FOR  
A NUCLEAR REACTOR CONTROL DRUM

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1. Report No. <b>NASA TM X-2297</b>		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle <b>ANALYTIC STUDY OF AN ELECTRIC STEPPING MOTOR DRIVE FOR A NUCLEAR REACTOR CONTROL DRUM</b>				5. Report Date <b>May 1971</b>	
				6. Performing Organization Code	
7. Author(s) <b>Arthur W. Kieffer</b>				8. Performing Organization Report No. <b>E-5788</b>	
				10. Work Unit No. <b>120-27</b>	
9. Performing Organization Name and Address <b>Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135</b>				11. Contract or Grant No.	
				13. Type of Report and Period Covered <b>Technical Memorandum</b>	
12. Sponsoring Agency Name and Address <b>National Aeronautics and Space Administration Washington, D. C. 20546</b>				14. Sponsoring Agency Code	
15. Supplementary Notes					
16. Abstract  This study analytically evaluates a stepping motor as a reactor control drum drive (drum weight is approximately 159 kg). Motor performance is evaluated for different load and motor conditions. Failure-free operation, where the motor rotates one step increment for each step command, is considered most important. Failures were less frequent when stepping was at a rate that allowed the system to come to rest between steps. A failure-free operating region is defined for this type of stepping rate.					
17. Key Words (Suggested by Author(s)) <b>Stepping motor Reactor control actuator Nuclear space power</b>				18. Distribution Statement <b>Unclassified - unlimited</b>	
19. Security Classif. (of this report) <b>Unclassified</b>		20. Security Classif. (of this page) <b>Unclassified</b>		21. No. of Pages <b>38</b>	
				22. Price* <b>\$3.00</b>	

# ANALYTIC STUDY OF AN ELECTRIC STEPPING MOTOR DRIVE FOR A NUCLEAR REACTOR CONTROL DRUM

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## SUMMARY

This study analytically evaluates the use of a stepping motor for positioning a reactor control drum (approximate weight, 159 kg). A spring is used for reactor scram. Motor performance is evaluated for two scram methods. In the first method the scram spring always remains coupled with the drum, while in the second method the scram spring is decoupled during motor operation and becomes coupled only for a reactor scram.

The study was performed parametrically to include future design changes and to find the best operating conditions. The operating conditions are satisfactory if the stepping motor can operate without a failure. A failure occurs when the motor fails to rotate one step increment for each step command. The results indicate that stepping failures are more likely for a coupled scram spring. When a failure occurs, the engaged spring usually rotates the drum to its scram position. A failure is less likely for single-step operation where stepping takes place at a rate that allows the actuator system to come to rest between steps. A failure-free operating region is defined for single-step operation.

## INTRODUCTION

A nuclear reactor heat source for generating 500 kilowatts of electric power in space is currently being studied at the Lewis Research Center. The reactor is considered to be compatible with either a Brayton or a Rankine cycle. The reactor is of the fast-spectrum type and is designed for 50 000 hours of operation at a power level of 2.17 megawatts. The reactor core consists of cylindrical fuel pins which are cooled by lithium. Six control drums are located at the periphery of the core as shown in figure 1. The drums are partially loaded with nuclear fuel and are rotated for the purpose of

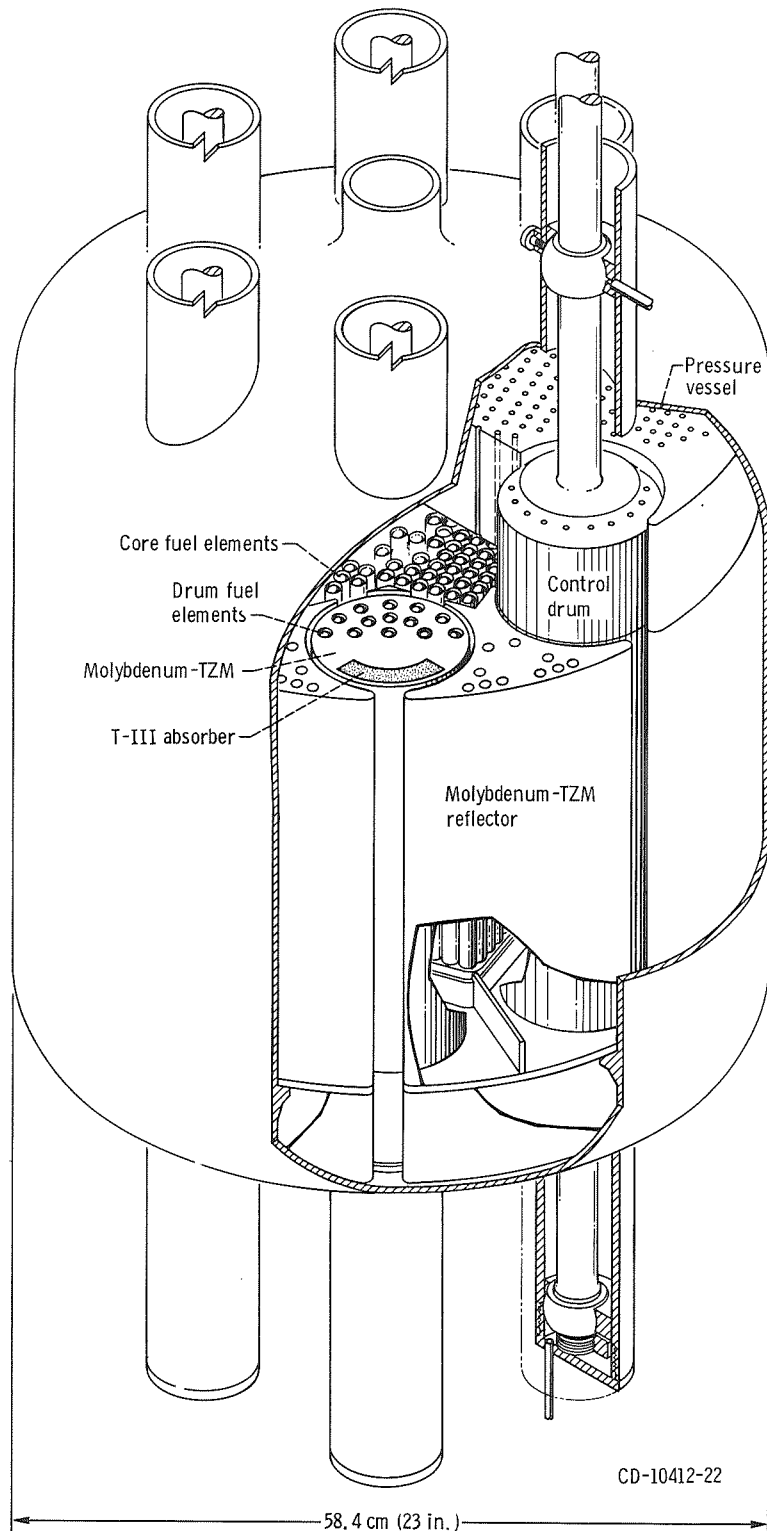


Figure 1. - Nuclear powerplant reactor.

changing the fuel geometry (reactivity) of the core. In this manner the reactor power is regulated.

An electric stepping motor is being considered for positioning each control drum. The motor is operated by step commands. For each command, the motor rotates a discrete increment. The stepping motor has several features that are desirable for our application. Gradual changes in drum position can be made with a slow stepping rate, and the drum position can be determined from the number of step commands. The inherent reliability of the stepping motor is perhaps its principal advantage for this application. It requires no rotor insulation, and relative to other electric motors it has fewer moving parts. Tests performed for the SNAP program (ref. 1) demonstrated a stepping motor actuator which operated continuously for 10 000 hours in a high-temperature nuclear environment. The reactors in SNAP, Phoebus (ref. 2), and NERVA (ref. 3) also use a stepping motor drive as a control actuator.

There are a great variety of stepping motors in existence today. Their application generally requires fast stepping of small loads. They are categorized as either a variable reluctance or a permanent-magnet type. This analysis considers a permanent magnet motor with four stator poles and a bar magnet rotor. This type of motor was previously analyzed by David J. Robinson in reference 4. The main part of his analysis is performed by phase plane, where he investigates some motor stepping failures for a simplified frictionless load.

The purpose of this study is to investigate the performance of a stepping motor as a control drum drive for the 500-kilowatt space power reactor. This application presents a relatively high frictional load. A slow stepping rate is acceptable and positioning accuracy is stressed. The study was performed parametrically to determine the best motor torque and optimum stepping rates. Because the present control drum design is subject to change, a parametric study is also useful for predicting the effect of future design changes. The following parameters were varied: (1) the motor torque, (2) the motor stepping rate, and (3) the motor load conditions. The results can be extended to motors with more poles having similar torque characteristics and a similar stepping sequence.

Most important in our application is to avoid stepping failures, and the successful operation of the stepping motor depends on this. A failure occurs when a step command fails to move the rotor by a prescribed step increment. The emphasis of this study is therefore on finding operating conditions that contribute to a failure and making recommendations to avoid them. Several steps are necessary to produce any significant reactivity change. Consequently, the overshoot and oscillations that may accompany each step affect the reactor very little. The oscillations nevertheless are significant to the motor because they may cause a stepping failure.

Three types of failures are investigated. The first failure is motor stall caused by

insufficient torque. Another failure results from a step command rate which is too fast. Consequently, the rotor lags the stepping rate and loses synchronism. The last failure happens at a slower stepping rate, and is caused by rotor oscillations that accompany a step increment in an underdamped system. This failure happens when another step command is given during the oscillation.

The parametric study was conducted on the analog computer for an idealized motor. Some analytic results are also presented which define a failure-free operating region for single-step operations. Before these results are presented, torque plots are discussed which give insight to the stepping-motor operation. These plots are useful for predicting some aspects of the motor's performance.

## DESCRIPTION OF ACTUATOR SYSTEM

The actuator system considered in this study consists of the reactor control drum and its actuator. The control drum has a moment of inertia of 0.47 joule-second<sup>2</sup> and weighs 159 kilograms. The drum friction is expected to change during the 50 000-hour operating lifetime and its present value is only an estimate. For these reasons the frictional torque is included as one of the varied parameters. Both static and coulomb friction are considered. The coefficient of friction of each is assumed to be of equal value. Torque values that are considered representative range from approximately 6.8 to 40.7 joules.

Two important reactor safety requirements are (1) positive reactivity insertion must be limited to less than 8¢ per second (ref. 5); (2) a fast insertion of negative reactivity is required for reactor scram (approximately 50¢/sec for a 3-sec scram based on a control drum worth of 150¢ at design). The first requirement limits the drum velocity to less than 4 degrees per second. To meet the second requirement a scram spring is used to drive the drum during a reactor scram.

Two scram concepts are being considered in the reactor design. In one concept a spring acts on the drum at all times. When a reactor scram becomes necessary, the actuator disengages from the drum. This allows the spring to drive the drum to its scram position. In the other concept, the spring remains decoupled from the drum while the actuator is engaged. The reactor can be scrambled by disengaging the actuator and then coupling the spring to the drum.

Drum positioning to within  $0.1^{\circ}$  is adequate. A change of  $0.1^{\circ}$  represents a reactivity change of approximately 0.2¢ based on current drum design. This reactivity is expected to produce a transient change in reactor power of less than 1 percent and a steady-state change of less than 0.4 percent (based on projected data from ref. 6). A block diagram of the actuator system is shown in figure 2.

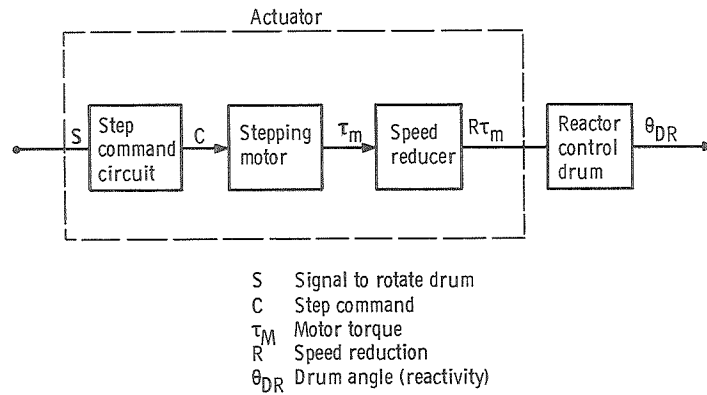


Figure 2. - Block diagram of actuator system.

Part of the actuator system is the stepping motor. It rotates in discrete increments or steps. The motor usually has several stator poles and a permanent magnet rotor arrangement. Energizing the stator windings creates a magnetic field which interacts with the permanent magnet rotor. For each step command the magnetic field rotates a discrete increment. As a result, the rotor also rotates because it tends to align with the field.

Figure 3 is a sketch of a four-pole stepping motor with a table of stator polarities for the entire stepping sequence. To illustrate a counterclockwise sequence, assume the stator is energized for step 0. The table shows that step 0 makes south poles of "A" and "B." These poles in turn create a resultant south pole at  $45^\circ$ . Similarly, poles "C" and "D" form a north pole at  $225^\circ$ . This polarity allows the rotor to be at rest with its poles aligned with opposite stator poles along a  $45^\circ$  diagonal, as shown in figure 3. Step command 1 shifts the stator polarities counterclockwise by  $90^\circ$ . In order to realign with the stator poles, the rotor must rotate from the  $45^\circ$  position shown in figure 3 to  $135^\circ$ . Step commands 2, 3, and 0 successively advance the stator polarities by an additional  $90^\circ$  for a total of one revolution. The polarities move in the opposite direction for a reverse sequence.

The stepping circuit which switches the stator polarities is represented by the block diagram of figure 2. A predetermined constant stepping rate results whenever the input signal "S" is positive. A negative "S" gives a constant stepping rate in the opposite direction.

The motor and the drum are coupled by a 900:1 speed reduction. This speed reduction was chosen to reduce the step increments of the drum to  $0.1^\circ$  and to increase the torque output of the motor.

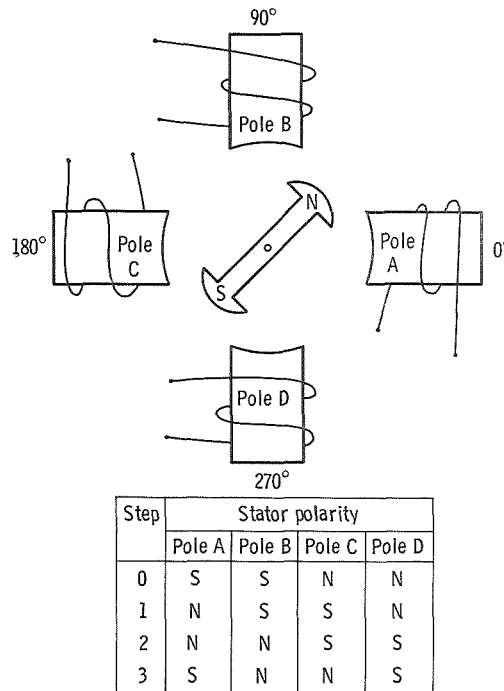


Figure 3. - Diagram of permanent-magnet stepping motor and table of stepping sequence.

## STEPPING MOTOR PERFORMANCE WITH A MOTOR LOAD

In the following discussion, torque curves are presented that give some insight into the stepping motor operation. The curves are useful for estimating the motor response with a specific load and for predicting possible stepping failures. The frictional torque, the spring torque, and the motor torque are the only torques considered. The motor torque is a function of the rotor position and is assumed to vary sinusoidally. The sum of these torques produces a net torque which determines rotor motion and thereby stepping motor performance.

We first show how a plot of net torque is obtained for a system with friction and a coupled spring by starting with a plot for a frictionless system. Only relative values of torque are important in this discussion and therefore the plots are for normalized torques. Figure 4(a) shows the net torque for a frictionless system with the spring decoupled and a maximum motor torque of 1.00. (All values of motor torque are given in terms of their peak sinusoidal values.) This plot is for step 0 and its net torque consists of motor torque only. This torque is shown as a sinusoidal function of the rotor north-pole positions. Positive torque induces counterclockwise (ccw) motion, and negative torque gives clockwise (cw) motion. The rotor can rest at a zero torque position shown at  $45^\circ$ . Rotation away from  $45^\circ$  is opposed by the net torque on either side of  $45^\circ$ .



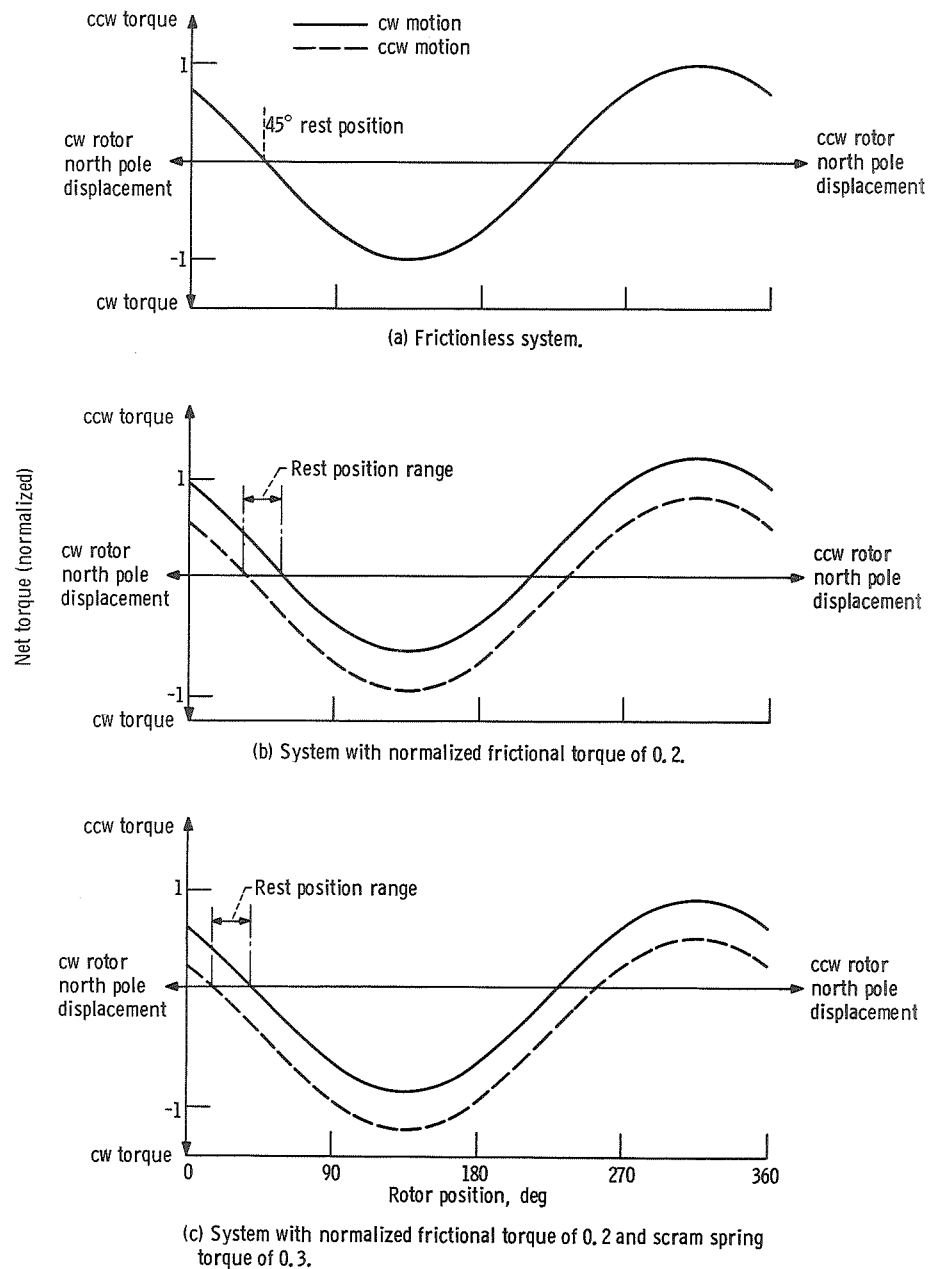


Figure 4. - Normalized net torque as function of rotor position for step 0 for normalized motor torque of 1.0.

At  $225^\circ$  the net torque is also zero but the net torque adjacent to  $225^\circ$  promotes rotation away from this position. This makes the  $225^\circ$  position unstable and therefore not a proper rest position.

Two curves of net torque are necessary to represent a frictional system. Frictional torque acts opposite to rotor motion. Therefore, when frictional torque is added to figure 4(a), a separate curve results for forward motion and another curve for reverse motion. The result is shown in figure 4(b), which is for a constant normalized frictional torque of 0.2.

For a frictional system, a range of rotor rest positions are possible. In figure 4(b) the range is centered at approximately  $45^\circ$  and is bounded by the zero torque points of both curves. An examination of figure 4(b) shows that no motion can originate in this rest position range. Counterclockwise motion requires a positive torque, and the curve for counterclockwise motion shows that positive torque is not available in that range. The curve for clockwise motion likewise shows that negative torque is not available for clockwise motion.

The coupled scram spring is represented by a constant torque which acts clockwise. The coupled scram spring can be included on the net torque plot by adding a constant negative torque to figure 4(b). Figure 4(c) shows the results for a normalized spring torque of 0.3. The addition of the spring torque shifts the rest position to the left of  $45^\circ$ .

Plots representing the system for the remaining steps are obtained by translating the plot of figure 4(c) to the right by  $+90^\circ$ ,  $+180^\circ$ , and  $+270^\circ$ . Plots for all four steps are shown in figure 5, where the motion of the rotor can be traced for the entire stepping sequence. The rotor is accelerated counterclockwise for positive net torque and clockwise for negative net torque. The rotor can come to rest and remain at rest at a position of zero net torque if the kinetic energy of the actuator system is also zero.

The kinetic energy of the actuator system is zero whenever the area traversed by the rotor in figure 5 is zero. A direct relation between the traversed area in figure 5 and the system kinetic energy can be demonstrated by starting with Newton's second law. When this law is extended to angular motion, we have the following equation (all symbols are defined in appendix A):

$$\tau_{\text{net}} = I\alpha_m = I \frac{d\omega_m}{dt} \quad (1)$$

Multiplying by  $d\theta_m$  gives

$$\tau_{\text{net}} d\theta_m = I \frac{d\omega_m}{dt} d\theta_m = I\omega_m d\omega_m \quad (2)$$

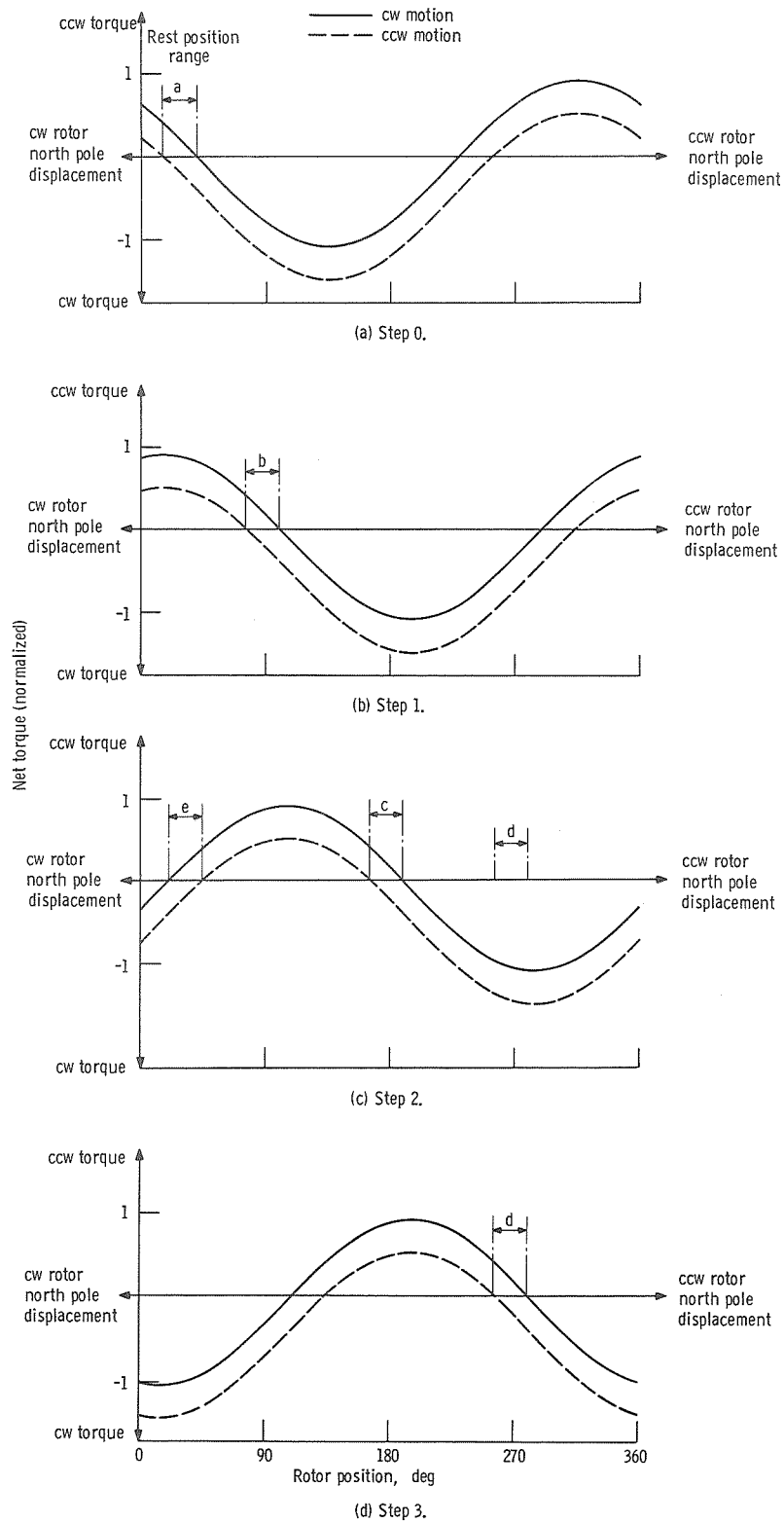


Figure 5. - Normalized net torque as function of rotor position for complete stepping sequence for normalized torques of 0.2 for friction, 0.3 for the scram spring, and 1.0 for the motor.

Integrating between the initial and final rotor angle gives

$$\int_{\theta_{m,0}}^{\theta_{m,f}} \tau_{\text{net}} d\theta_m = I \int_{\omega_{m,0}}^{\omega_{m,f}} \omega_m d\omega_m = \frac{1}{2} I \omega_{m,f}^2 - \frac{1}{2} I \omega_{m,0}^2 = \text{System kinetic energy} \quad (3)$$

But since the first term in equation (3) is the area traversed by the rotor in figure 5, we have thus proven this area to be equal to the system kinetic energy.

We can use relation (3) in conjunction with the net torque plot in figure 5 to determine graphically the rotor overshoot, the number of oscillations, and the final rotor rest position. For example, assume that the rotor is initially at rest at its step 3 rest position, designated by "d" in figure 5(d). Step command 2 makes the net torque negative at position "d," as shown in figure 5(c). Consequently, for step 2 the rotor moves to the left of "d" until it traverses a net area beneath the clockwise motion curve in figure 5(c). This occurs in the vicinity of  $100^\circ$ , where the rotor stops momentarily. It then moves counterclockwise because the net torque is positive at  $100^\circ$ . The rotor continues in this direction until it covers a net area of zero beneath the curve for counterclockwise motion in figure 5(c). The rotor continues to oscillate in this manner. Each cycle decreases in amplitude in proportion to the friction of the system. The rotor finally comes to rest when the net torque and the traversed area become zero simultaneously. This occurs in the rest position range designated "c."

The rotor motion is similar for other step commands. When step commands are given after the rotor comes to rest, it will be referred to as "single-step operation." With some additional effort, figure 5 can also be used to determine multistep operation. For this case the rotor is not at rest at the time of a step command and the net area is therefore not zero. Consequently, for multistep operation the leftover area must be carried into the next step as an initial condition.

Plots like figure 5 can indicate how some parameters affect the system. Higher friction, for example, will show more separation between the curves of clockwise and counterclockwise motion in figure 5. This widens the range of the rest positions. The rotor may therefore come to rest at various positions and, consequently, the step increments may be less uniform. By tracing the rotor motion in the manner described previously, its response to a step command can also be estimated.

Conditions for a stepping failure can be detected from figure 5. To illustrate, consider the previous examples which traced the rotor motion from step 3 to step 2. For this example the rotor starts from "d" in figure 5(c) and moves left. A stepping failure occurs if the area cannot become zero before "e" in figure 5(c). In this event, the rotor may either come to rest at "e" or continue past "e." If the rotor moves past "e," the

net area can never become zero and the rotor continues to move clockwise. In this event the motor has lost control and the scram spring takes the drum to its scram position.

A failure which moves the drum to its scram position is avoided if the scram spring is decoupled. Stepping failures are then less serious and also less frequent. Such failures end with the drum out of position by one or, at most, several step increments. However, stepping failures usually reoccur and therefore failures in general are undesirable. The probability of a failure is reduced for single-step operation because for multistep operation the traversed area in figure 5 may accumulate and cause a failure. Single-step operation avoids this accumulation because the area is zero prior to each step command.

## RESULTS AND DISCUSSION

The successful operation of the stepping motor as a control drum drive depends on its ability to operate without a stepping failure. Stepping failures are undesirable because the worst cases make drum positioning impossible. At best, stepping failures make drum positioning more time consuming because more than the usual number of step commands are necessary to bring the drum to its proper position. The drum position also cannot be determined from the number of step commands. A stepping failure with the scram spring decoupled causes the drum to be out of position by no more than several step increments. Although this positioning error is small, the operating conditions that caused one failure usually cause additional failures. A stepping failure is most severe with the scram spring coupled because this usually moves the drum to its scram position.

Various operating conditions were investigated to determine where stepping failures occur and where the motor is least likely to fail. Analytic results from appendix B show that for single-step operation the following conditions can lead to a stepping failure:

(1) Whenever the magnitude of the spring torque plus the magnitude of the frictional torque is equal to or greater than 0.707 times the motor torque ( $\tau_{M, \max}$ ), the motor stalls. With the coupled scram spring, the motor stalls only when moving against the spring, or counterclockwise.

(2) Whenever the magnitude of the spring torque minus the magnitude of the frictional torque is equal to or greater than 0.308 times the motor torque, a failure may occur while moving clockwise. But this failure only happens with a coupled scram spring.

The two conditions leading to a stepping failure during single-step operation are illustrated by figure 6. The plot shows the minimum motor torque necessary for failure-free bidirectional operation. This motor torque is plotted as a function of frictional torque for scram spring torques capable of 1-, 3-, and 6-second scram times. For bi-

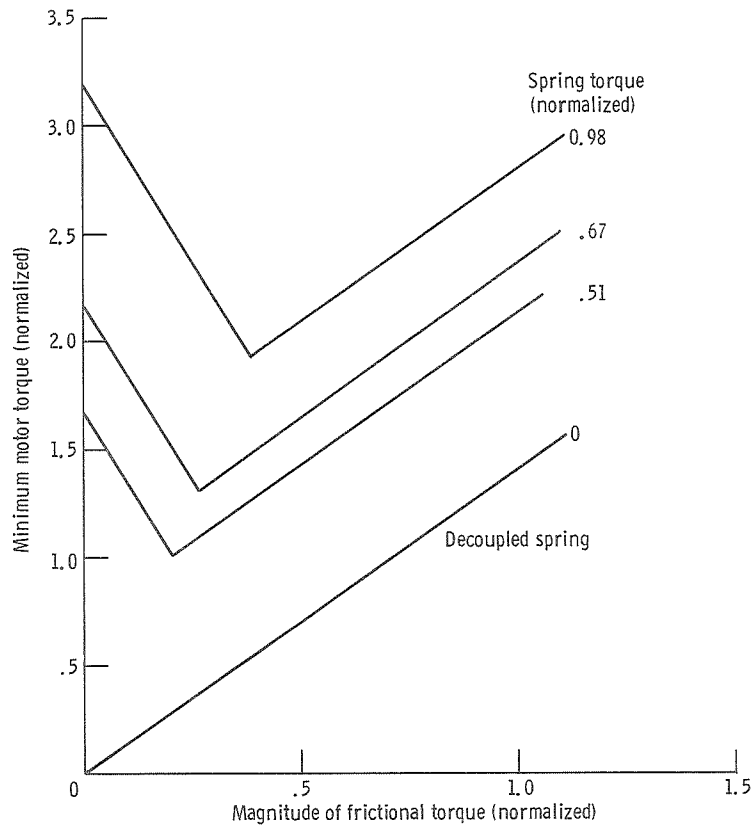


Figure 6. - Minimum motor torque required for single-step operation for various frictional and spring torques.

directional rotation, failure-free, single-step operation is possible in the region located above the line of minimum motor torque. For example, figure 6 shows that for a spring torque of 0.67 and a frictional torque of 0.50, the least motor torque necessary for failure-free, single-step operation is approximately 1.67.

For multistep operation, two additional conditions contribute to a stepping failure:

- (1) When the command rate is too fast, a failure occurs because the rotor loses synchronism.
- (2) A failure may also occur when a step command is given while the actuator system is oscillating. (Such oscillations usually occur after each step increment in an underdamped system.)

A study was conducted on the analog computer to determine the stepping motor performance in detail and the conditions that produce stepping failures. Significant parameters were varied about their present design value to determine their effect on the motor. These parameters are listed in the following table which also includes their reference value and the range of variation:

Parameter	Range of values		
	Low	Reference	High
Normalized frictional torque (torque needed to overcome friction)	0.11	0.43	0.65
Normalized motor torque, $\tau_{M, N}$	.67	1.00	2.22
Normalized motor torque, $\tau_{M, N}$ (coupled spring)	1.56	2.22	2.22
Normalized scram spring torque	.51 (6 sec <sup>a</sup> )	.67 (3 sec <sup>a</sup> )	.98 (1 sec <sup>a</sup> )
Backlash, deg	.0	.0	.1

<sup>a</sup>Scram time with frictional torque at 0.43. Torques are normalized to 61 J.

The motor torque values in the table refer to the torque obtained from the speed reduction and pertain to the maximum part of the sinusoidal function. When the spring is engaged, the additional load makes a higher motor torque necessary. Therefore, a different reference motor torque is given for this case in the table. All torque values are normalized to 61 joules.

Several simplifying assumptions are made throughout this study. It is assumed that the motor torque is produced by a uniform magnetic field (stator poles) acting on a thin bar magnet (rotor). This makes the motor torque a sinusoidal function of the rotor angle. Electric and magnetic transients that accompany a change in stator polarity are neglected. The counter electromotive force induced by rotor motion is also neglected. Some viscous damping is included for the rotor but not for the drum.

## Variations of Motor Torque and Friction with a Decoupled

### Scram Spring and No Backlash

The plots of figure 7 show the effect of different motor torques on the drum response. Each plot in figure 7 gives the drum position as a function of time. The plots are for a frictional torque of 0.43 with the scram spring decoupled. The triangles on the abscissa of each plot indicate the time of each step command. Each step increment corresponds to a drum rotation of approximately  $0.1^\circ$ . The drum response is shown in figures 7(a) to (d) for normalized motor torques of 0.67, 1.00, 1.33, and 2.22, respectively. For motor torques of 0.67 and 1.00 the response is overdamped. It is underdamped for the remaining two torques of 1.33 and 2.22.

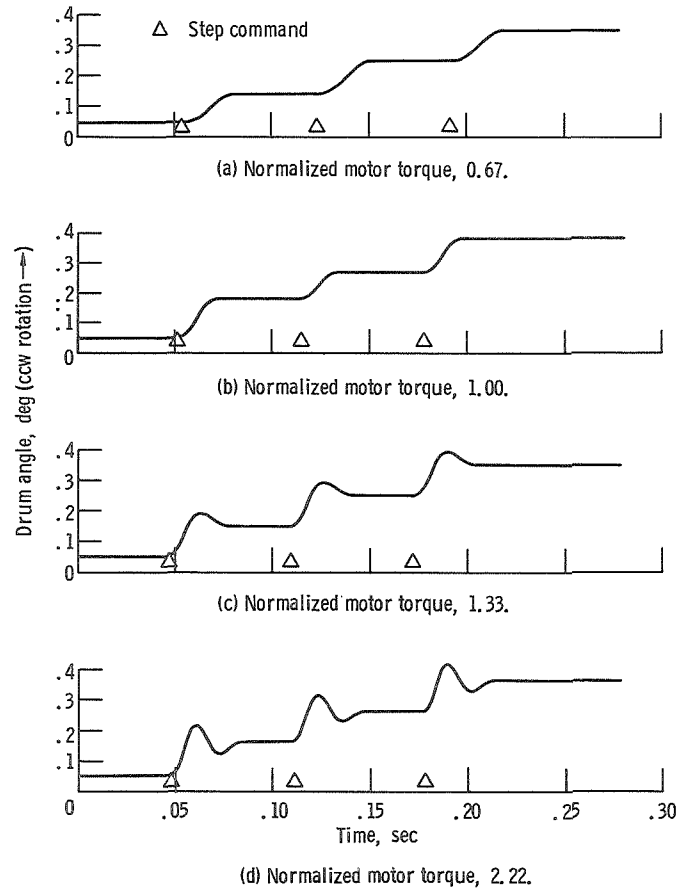


Figure 7. - Time response of control drum to three step commands for various motor torques, normalized frictional torque of 0.43, and a decoupled scram spring.

The effect of friction on the drum response is shown in figure 8. Plots for normalized frictional torques of 0.22, 0.43, and 0.65 are shown in figures 8(a) to (c), respectively. The motor torque for these plots is at 1.00. The plot for the frictional torque of 0.22 shows some oscillations. The response for frictional torques of 0.43 and 0.65 is overdamped.

The preceding results show that a high motor torque relative to friction causes oscillations after each step. These oscillations are undesirable for multistep operation because they may cause a stepping failure. The oscillations are also undesirable for single-step operation because they make a low stepping rate necessary. For the reference system whose parameters are listed in the previous table, a normalized motor torque of 1.00 was found to be the best value. (It is therefore referred to as the reference value.) This reference torque is low enough to eliminate the undesirable oscillations and also is at a satisfactory level above the necessary minimum established in



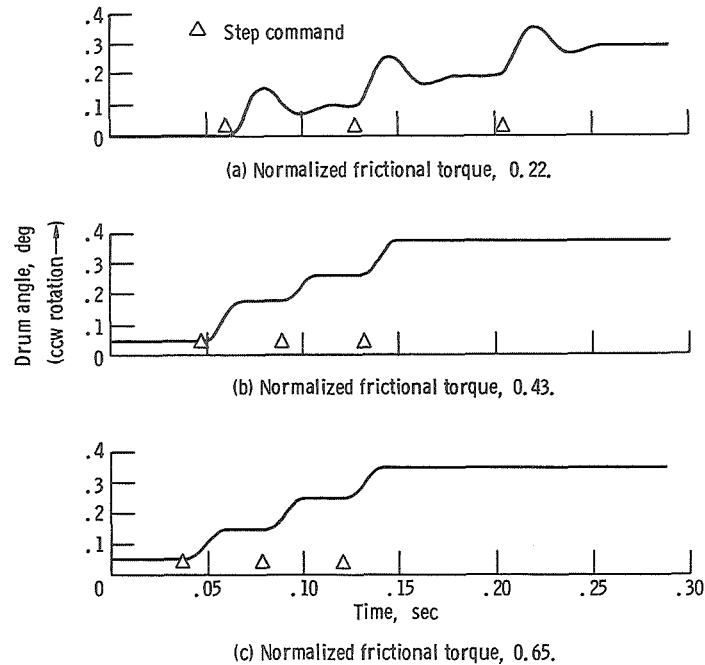


Figure 8. - Time response of control drum for various frictional torques with normalized motor torque of 1.00 and a decoupled scram spring.

figure 6. Drum response for the reference system with this motor torque is shown in figure 8(b).

### Results with the Scram Spring Coupled and No Backlash

The response with a coupled scram spring was studied for normalized spring torques of 0.51, 0.67, and 0.98. The normalized frictional torque for this part of the study is at 0.43. A normalized motor torque of 2.22 was used. The high motor torque was necessary to accommodate the cases with high scram spring torques. The response for all three cases is similar and therefore only the plot for the 0.67 case is presented. Figure 9(a) shows the results for three step commands against the spring force and one command with the spring force. Overshoot occurs for all four steps.

An unsuccessful attempt toward an overdamped response was made by reducing the

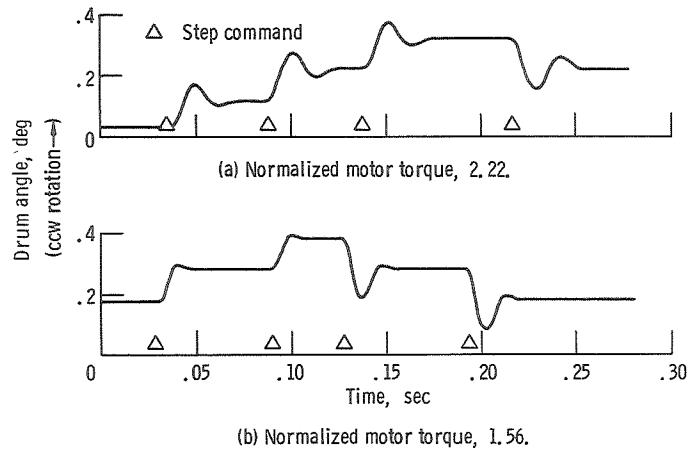


Figure 9. - Time response of control drum for normalized frictional torque of 0.43 with a coupled scram spring having normalized torque of 0.67.

motor torque. Only the response for the counterclockwise direction improved. The results are shown in figure 9(b) for a motor torque of 1.56. To enable a reactor scram, the scram spring torque must always be higher than the frictional torque, and therefore an overdamped response is not possible.

## System Response with Backlash

The coupling which connects both the speed reducer and the drum may be a source of backlash. A probable backlash value of  $0.1^\circ$  was chosen to investigate the effect of backlash with a coupled and a decoupled scram spring.

Decoupled scram spring. - The plots contained in figure 10 show the response for a system with backlash when the scram spring is decoupled and the system parameters are at their reference value. Both drum and rotor move independently in the backlash region. A plot for each response is therefore presented. A comparable plot without backlash was previously presented in figure 7(b).

Figure 10(a) shows the drum position plotted against time, and figure 10(b) shows the corresponding rotor position. The rotor is shown to be initially at rest at  $45^\circ$  in the center of the backlash region decoupled from the drum. At approximately  $t = 0.035$  second, a step command advances the rotor toward  $90^\circ$ . At approximately  $90^\circ$  the rotor has moved out of the backlash region and becomes coupled to the drum. At that time the added load of the drum reduces the rotor speed. The change in speed can be noticed in figure 10(b), where a change in the slope of the curve occurs at approximately 0.04 second. The drum then begins to move as shown in figure 10(a). During the next 0.01 second the rotor moves the drum to a position of approximately  $0.11^\circ$ , where it then

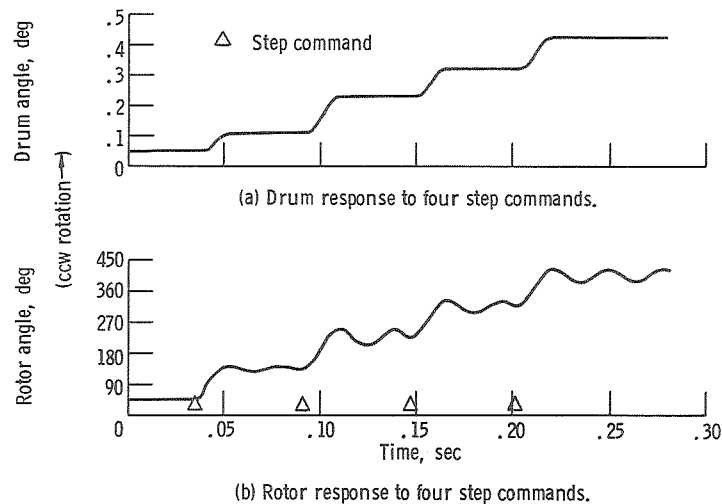


Figure 10. - Time response of reference actuator system with decoupled scram spring and 0.1° backlash.

comes to rest. The rotor, however, reverses and moves into the backlash region where it continues to oscillate. It would eventually come to rest at  $135^{\circ}$  if another step command were not given at  $t = 0.09$  second. Two more step commands then follow.

Coupled scram spring. - Figure 11 shows the results of 0.1° backlash with the scram spring coupled to the drum. The system parameters are again at their reference value (motor torque reference value updated to 2.22 to account for the spring). A comparable plot without backlash was previously presented in figure 9(a).

Figure 11(a-1) shows the drum position plotted against time for stepping counter-clockwise against the spring. The corresponding rotor position is plotted in figure 11(a-2). The spring keeps both drum and rotor coupled while at rest and for the most part of the rotation. Therefore, figures 11(a-1) and (a-2) show similar response.

Figures 11(b-1) and (b-2) show three clockwise steps for the drum and the rotor respectively. The first step is a typical example of the system response. Immediately following the step command, the rotor decouples from the drum and moves into the backlash region. The spring then forces the drum to follow the rotor. Figure 11(b-2) shows that the rotor eventually overshoots its intended position and, on its return, again couples to the drum at approximately  $t = 0.037$  second while both move in opposite directions. Subsequently, both drum and rotor move together and each shows a similar response. Eventually, the rotor comes to rest at approximately  $300^{\circ}$  and the drum at approximately  $0.28^{\circ}$ .

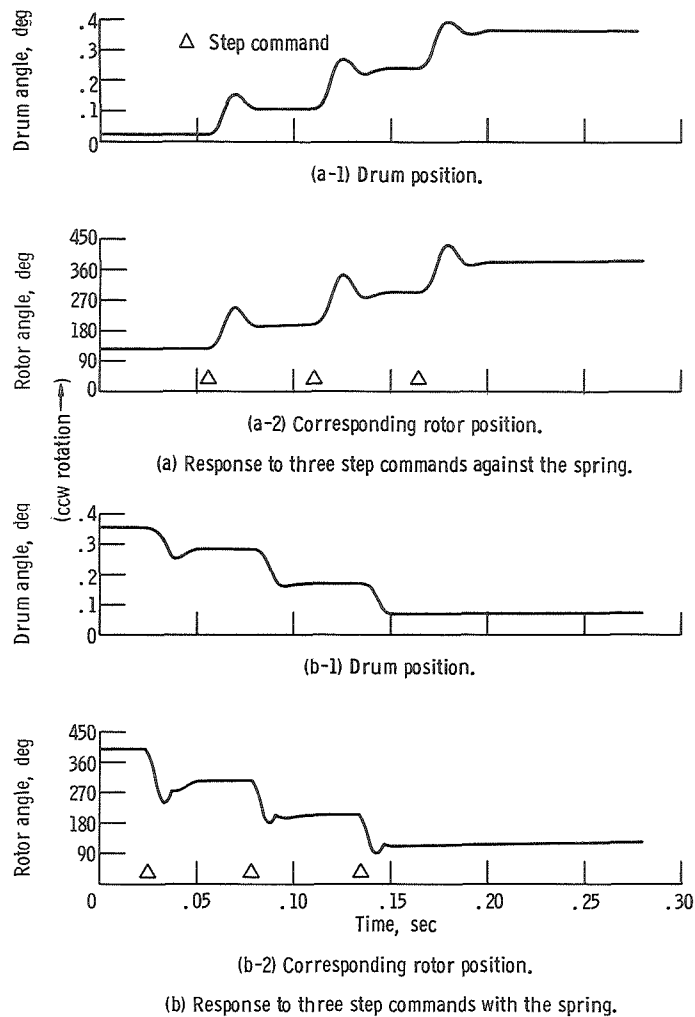


Figure 11. - Time response of reference actuator system with coupled scam spring and 0.1° backlash.

## Cases of Stepping Failures

Examples of stepping failures during multistep operation were demonstrated on the analog computer. The type of stepping failure that is first presented shows a failure from a high stepping rate.

Figure 12 shows the drum response for three different stepping rates. The plots were obtained with the spring decoupled and with the motor and frictional torques at their reference values. The first plot, figure 12(a), shows the response for single-step operation. It shows satisfactory operation for a stepping rate of 2 degrees per second. The operation also remained satisfactory for higher stepping rates up to and including a stepping rate of 13.6 degrees per second - well beyond the safe operating

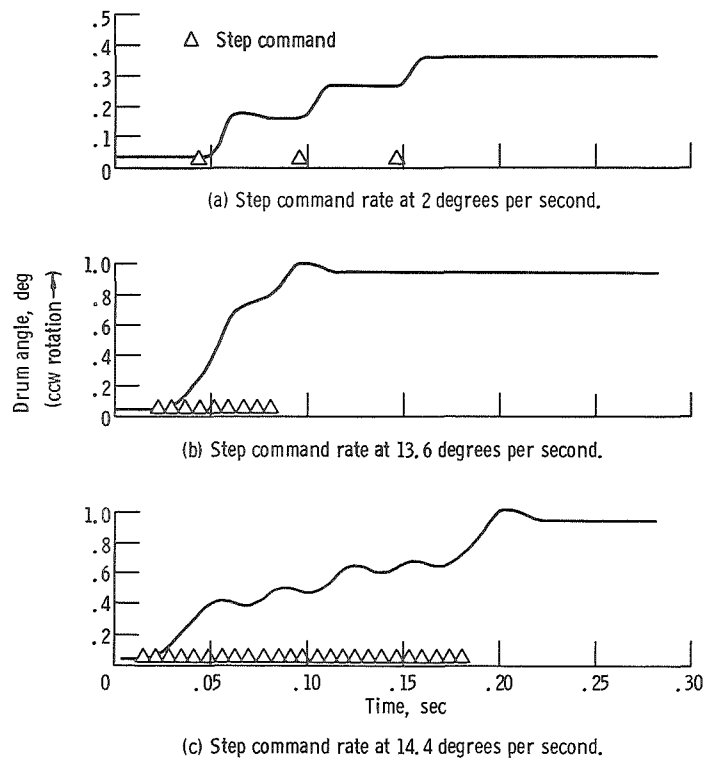


Figure 12. - Time response of control drum for reference system with decoupled scram spring.

rate of 4 degrees per second recommended in reference 4. The response for a stepping rate of 13.6 degrees per second is shown in figure 12(b). In this case a total of nine step commands were given and the drum advanced approximately  $0.9^\circ$  or nine steps; no stepping failure therefore takes place. If the stepping rate is further increased, a stepping failure occurs. This failure is shown in figure 12(c) for a stepping rate of 14.4 degrees per second. For this case the drum advances a total of only nine steps for 25 step commands.

For our application the stepping rate will never be as high as 14.4 degrees per second. However, a failure can also occur at a lower stepping rate. This type of failure occurs because of the rotor oscillations that accompany a step increment in an underdamped system. The failure may happen when a step command is given during the undershoot part of the oscillation. Figure 13 shows a stepping failure due to this condition where a step command is given during the undershoot part of the oscillation. The plot is for a system with a normalized motor torque of 1.00, a normalized frictional torque of 0.11, and a stepping rate of 2.27 degrees per second (or 22.7 steps per second). The scram spring is decoupled and backlash is not considered. The results show that the drum response is irregular and oscillatory and that several stepping failures occur. The first failure

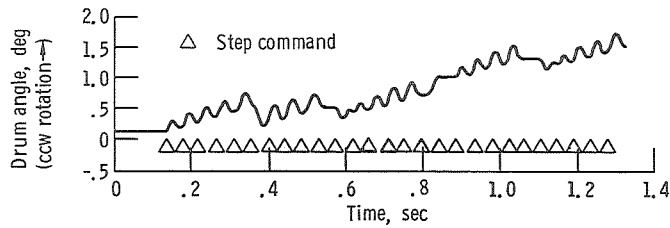


Figure 13. - Time response of control drum with motor torque of 1.00, frictional torque of 0.11, stepping rate of 2.27 degrees per second, with decoupled scram spring.

takes place during the sixth step command. It occurs when a command is given during the undershoot cycle of the drum oscillation at approximately  $t = 0.35$  second. Figure 13 shows three more stepping failures at approximately 0.55, 0.80, and 1.05 seconds.

A stepping failure is much more severe when the scram spring is coupled. The spring then forces the drum to its  $0^\circ$ , or scram, position. Figure 14 illustrates this type of a stepping failure. The scram spring torque is 0.67, the frictional torque 0.22, and the motor torque 2.22. The stepping rate is 2 degrees per second. The failure

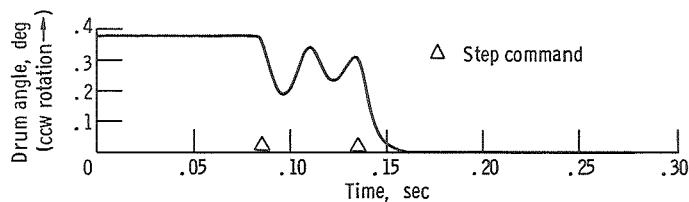


Figure 14. - Time response of control drum to two step commands for moving with spring force at rate of 2 degrees per second. Spring torque, 0.67; motor torque, 2.22; frictional torque, 0.22.

occurs during the second step command starting at approximately  $t = 0.135$  second.

## Allowable Stepping Rates for Single-Step Operation with a Decoupled Scram Spring

Both multistep operation and a coupled scram spring appear to be troublesome. Single-step operation appears more desirable except for the slow stepping rate. This section therefore investigates stepping rates for single-step operation. For this investigation the motor torque is 1.00, the spring is decoupled, and backlash is zero.

Figure 12(a) showed that single-step operation is possible for a stepping rate of 2 degrees per second when the frictional torque is at the reference value of 0.43. Since this frictional value is preliminary and since it will quite probably change, the stepping rate for single-step operation was further investigated for different frictional torques. The results are shown in figure 15. This plot shows the maximum stepping rates allowed for single-step operation for various normalized frictional torques. The plot shows that a stepping rate of up to approximately 6.5 degrees per second is possible for the reference system. If the stepping rate is selected to be 2 degrees per second (which

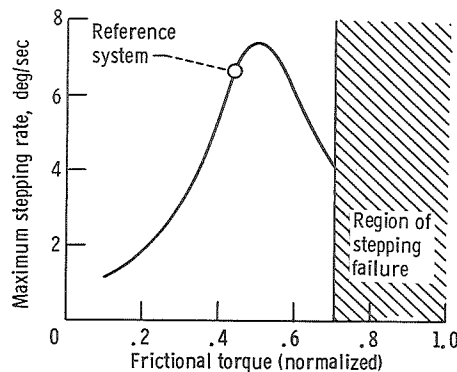


Figure 15. - Maximum stepping rates for single-step operation with decoupled scram spring and normalized motor torque of 1.00.

at present seems a reasonable rate for reactor control), figure 15 shows that single-step operation is possible for normalized frictional torques from approximately 0.2 to 0.707.

## CONCLUDING REMARKS

A permanent-magnet stepping motor was evaluated as a reactor control drum drive, assuming an idealized motor. The successful operation of this motor depends on its ability to operate without a stepping failure. A stepping failure occurs when a step command fails to move the rotor a prescribed increment.

The main source of a stepping failure was from a coupled scram spring. A coupled scram spring required a higher motor torque. The higher torque makes the system underdamped and therefore more susceptible to stepping failures. During a failure the scram spring forces the drum to its scram position. It is therefore recommended that the scram spring be decoupled during normal operation and be coupled only for scram purposes.

The second most frequent source of a failure was from multistep operation where the actuator system does not come to rest between steps. This failure can happen with a decoupled or a coupled scram spring. Since multistep operation appears troublesome, it is desirable to operate at a stepping rate that allows the system to come to rest between steps (single-step operation). A failure-free operating region was found for single-step operation for both a decoupled scram spring and a coupled scram spring. A certain minimum motor torque is necessary to operate in the region. A motor torque which is considerably higher than the minimum should not be used because the system response becomes underdamped and a slower stepping rate may therefore be necessary for maintaining single-step operation.

Single-step operation is recommended. Single-step operation is possible with a stepping rate at 2 degrees per second for the reference system with a decoupled spring and no backlash. The 2 degree-per-second rate also allows single-step operation for frictional torques that differ considerably from the reference system.

Backlash should be avoided because when the rotor is decoupled from the load its response is underdamped. Frequent coupling and decoupling between motor and load was shown to occur because of backlash. The stress from this coupling process may be a significant problem.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, February 18, 1971,  
120-27.



## APPENDIX A

### SYMBOLS

C	step command	$\omega, \dot{\theta}$	angular velocity, rad/sec
D	viscous damping, J-sec	Subscripts:	
f	coefficient of friction	C	coulomb
g	gravitational acceleration, m/sec <sup>2</sup>	DR	drum
I	moment of inertia, J-sec <sup>2</sup>	F	friction
M	drum mass, kg	f	final
R	speed reduction	K	spring
r	length of moment arm, m	L	load after speed reduction
S	signal to rotate drum	l	load before speed reduction
t	time, sec	M	motor after speed reduction
$\alpha, \ddot{\theta}$	angular acceleration, rad/sec <sup>2</sup>	m	motor before speed reduction
$\Delta$	step command	max	maximum
$\delta$	backlash, rad	N	normalized
$\theta$	angular rotation, rad	S	static
$\tau$	torque, J	0	initial
$\tau_{net}$	resultant normalized torque at rotor		

## APPENDIX B

### DERIVATION OF SAFE OPERATING LIMITS FOR SINGLE-STEP OPERATION

Two types of failures are possible for single-step operation. One occurs when the motor torque is too low causing the motor to stall. For a coupled scram spring, stall occurs only when stepping counterclockwise against the spring. The other type of failure happens only when the scram spring is coupled when stepping clockwise. In this case the spring torque overpowers the motor and moves the drum to its scram position.

The first part of this appendix finds the minimum torque necessary to step counterclockwise against the spring and the second part does the same for stepping clockwise with the spring. The derivation assumes the rotor to be at rest at a position of zero net torque prior to a step (single-step operation). Friction is positive for counterclockwise motion and negative for clockwise motion. Friction permits a range of possible rest positions. In both parts the rotor is assumed to be initially at rest at a point in the range that makes a stepping failure most likely. Consequently, the analysis gives a sufficient but not a necessary motor torque requirement for avoiding a failure. Backlash is not considered. The equations in this appendix are special cases and were derived from the more general equations appearing in appendix C.

#### Minimum Torque for Counterclockwise Stepping

In this part we show that stepping against the spring is not possible if the magnitude of the spring torque plus the magnitude of the frictional torque is greater than  $0.707 \tau_{M, \max}$ . (This torque requirement was previously derived by Robinson in ref. 4 but for different load conditions.)

Prior to a step the rotor is assumed to be at rest at a position of zero net torque at  $\theta_{m, 0}$ . This gives the following torque balance at the rotor:

$$\tau_M + \tau_F + \tau_K = 0 \quad (B1)$$

Since  $\tau_K = -|\tau_K|$ , and for counterclockwise rotation  $\tau_F = -|\tau_F|$ , we can write equation (B1) as follows:

$$\tau_M = |\tau_F| + |\tau_K| \quad (B2)$$

Initially, the stator is arbitrarily assumed to be energized for step 0. The motor torque

for step 0 is given in appendix C; specifically, at  $\theta_{m,0}$  the motor torque is

$$\tau_M = -\tau_{M, \max} \sin\left(\theta_{m,0} - \frac{\pi}{4}\right) \quad (B3)$$

Substituting this equation into equation (B2) gives

$$-\tau_{M, \max} \sin\left(\theta_{m,0} - \frac{\pi}{4}\right) - |\tau_F| - |\tau_K| = 0 \quad (B4)$$

Let

$$|\tau_F| + |\tau_K| \geq +0.707 \tau_{M, \max} \quad (B5)$$

Then equation (B4) shows that the rotor must be at a rest position  $\theta_{m,0}$  located between  $-\pi/2$  and 0, or

$$-\frac{\pi}{2} \leq \theta_{m,0} \leq 0 \quad (B6)$$

To move the rotor from  $\theta_{m,0}$  against the spring, step command 1 must be given. Step 1 changes the motor torque to the following value at  $\theta_{m,0}$ :

$$\tau_M = -\tau_{M, \max} \sin\left(\theta_{m,0} - \frac{3\pi}{4}\right) = \tau_{M, \max} \sin\left(\theta_{m,0} + \frac{\pi}{4}\right) \quad (B7)$$

If stepping against the spring is to take place, the sum of the torques must be greater than zero for step 1 or

$$\tau_{M, \max} \sin\left(\theta_{m,0} + \frac{\pi}{4}\right) - |\tau_F| - |\tau_K| > 0 \quad (B8)$$

Since from equation (B5)  $|\tau_F| + |\tau_K| \geq 0.707 \tau_{M, \max}$  then

$$\tau_{M, \max} \sin\left(\theta_{m,0} + \frac{\pi}{4}\right) - 0.707 \tau_{M, \max} > 0 \quad (B9)$$

But equation (B9) cannot be satisfied for  $-\pi/2 \leq \theta_{m,0} \leq 0$ . Consequently, no stepping takes place for  $|\tau_F| + |\tau_K| \geq 0.707 \tau_{M,\max}$ . To avoid this stepping failure, the opposite must then be true, or

$$0.707 \tau_{M,\max} > |\tau_F| + |\tau_K| \quad (\text{B10})$$

Equation (B10) applies when stepping against the spring. When the spring is decoupled,  $\tau_K$  is zero and equation (B10) then applies for stepping in either direction.

### Minimum Torque for Clockwise Stepping

When stepping clockwise with the spring, the spring torque may cause a stepping failure by forcing the drum to its scram position. This failure occurs when during a step increment the rotor overshoots its intended position, the first zero net torque position, and rotates up to or beyond the next zero net torque position (for more detail see the main text section STEPPING MOTOR PERFORMANCE WITH A MOTOR LOAD). We will show that this type of failure can be avoided if the motor torque satisfies the following inequality:

$$0.308 \tau_{M,\max} > |\tau_K| - |\tau_F| \quad (\text{B11})$$

For this derivation let us start with the equation for combined drum and rotor motion and neglect backlash and viscous damping. We then have

$$I\ddot{\theta}_m = \tau_m + \frac{\tau_K}{R} + \frac{\tau_F}{R} \quad (\text{B12})$$

where  $I = I_m + I_{DR}/R^2$ . For clockwise movement we can express equation (B12) as follows:

$$I\ddot{\theta}_m = \tau_m - \left| \frac{\tau_K}{R} \right| + \left| \frac{\tau_F}{R} \right| \quad (\text{B13})$$

For this proof, we arbitrarily select step 0 for an initial condition. This gives the following equation for the motor torque:

$$\tau_m = -\tau_{m, \max} \sin\left(\theta_m - \frac{\pi}{4}\right) \quad (\text{B14})$$

The rotor is assumed to be initially at rest, and this position is designated by  $\theta_{m,0}$ . The acceleration  $\ddot{\theta}_m$  is then zero and equation (B13) can be written as follows for  $\theta_m = \theta_{m,0}$ :

$$0 = -\tau_{m, \max} \sin\left(\theta_{m,0} - \frac{\pi}{4}\right) - \left|\frac{\tau_K}{R}\right| + \left|\frac{\tau_F}{R}\right| \quad (\text{B15})$$

To move the rotor from  $\theta_{m,0}$  clockwise in the direction of the spring, step command 3 must be given. For this step the motor torque becomes

$$\tau_m = -\tau_{m, \max} \sin\left(\theta_m + \frac{\pi}{4}\right) \quad (\text{B16})$$

A failure occurs for step 3 if the rotor overshoots up to the second zero net torque position designated by  $\theta_{m,f}$ . At  $\theta_{m,f}$  the net torque is zero and therefore  $\ddot{\theta}_m$  is also zero. By letting  $\ddot{\theta} = 0$  and substituting equation (B16) into (B13) we have for  $\theta_m = \theta_{mf}$ :

$$0 = -\tau_{m, \max} \sin\left(\theta_{m,f} + \frac{\pi}{4}\right) - \left|\frac{\tau_K}{R}\right| + \left|\frac{\tau_F}{R}\right| \quad (\text{B17})$$

Then using equation (B15) to eliminate  $\tau_K$  and  $\tau_F$ , we obtain the following relation from equation (B17):

$$\sin\left(\theta_{m,f} + \frac{\pi}{4}\right) = \sin\left(\theta_{m,0} - \frac{\pi}{4}\right) \quad (\text{B18})$$

This equation can be solved for  $\theta_{m,f}$  in terms of  $\theta_{m,0}$ . There are an infinite number of solutions, but only solutions between 0 and  $2\pi$  are of interest. This limits the number to the two following solutions:

$$\theta_{m,f} = \theta_{m,0} - \frac{\pi}{2} \quad (\text{B19})$$

$$\theta_{m,f} = -\theta_{m,0} - \pi \quad (\text{B20})$$

The first solution is discarded because it gives the first zero net torque position. The failure takes place at the second zero net torque position given by equation (B20).

The condition that allows the rotor to reach  $\theta_{m,f}$  and thus produce a failure is determined next. As the rotor moves from its rest position  $\theta_{m,0}$  toward its second zero net torque position  $\theta_{m,f}$ , it will continue to move until the system kinetic energy becomes zero. If the kinetic energy does not become zero before  $\theta_{m,f}$ , the rotor will move up to or beyond  $\theta_{m,f}$  and a failure will occur. It can be shown from equation (B13) that with the rotor starting from  $\theta_{m,0}$  the kinetic energy for step 3 at any angle  $\theta_m$  is given by the following equation:

$$\begin{aligned} \text{Kinetic energy} = \frac{-I\omega^2}{2} = & -\tau_{m,\max} \cos\left(\theta_m + \frac{\pi}{4}\right) + \tau_{m,\max} \cos\left(\theta_{m,0} + \frac{\pi}{4}\right) \\ & + \left( \left| \frac{\tau_F}{R} \right| - \left| \frac{\tau_K}{R} \right| \right) (\theta_{m,0} - \theta_m) \end{aligned} \quad (\text{B21})$$

If the kinetic energy becomes zero at (or after)  $\theta_{m,f}$ , a stepping failure takes place and for this condition equation (B21) gives the following for  $\theta_m = \theta_{m,f}$ :

$$0 = -\tau_{m,\max} \cos\left(\theta_{m,f} + \frac{\pi}{4}\right) + \tau_{m,\max} \cos\left(\theta_{m,0} + \frac{\pi}{4}\right) + \left( \left| \frac{\tau_F}{R} \right| - \left| \frac{\tau_K}{R} \right| \right) (\theta_{m,0} - \theta_{m,f}) \quad (\text{B22})$$

Substituting equations (B20) and (B15) into (B22) give

$$\begin{aligned} 0 = & -\tau_{m,\max} \cos\left(-\theta_{m,0} - \frac{3\pi}{4}\right) + \tau_{m,\max} \cos\left(\theta_{m,0} + \frac{\pi}{4}\right) \\ & + \tau_{m,\max} \sin\left(\theta_{m,0} - \frac{\pi}{4}\right) (2\theta_{m,0} + \pi) \end{aligned} \quad (\text{B23})$$

Substituting into equation (B23) the identities

$$\left. \begin{aligned} -\cos\left(-\theta_{m,0} - \frac{3\pi}{4}\right) &= \sin\left(\theta_{m,0} + \frac{\pi}{4}\right) \\ \sin\left(\theta_{m,0} - \frac{\pi}{4}\right) &= -\cos\left(\theta_{m,0} + \frac{\pi}{4}\right) \end{aligned} \right\} \quad (\text{B24})$$

and rearranging, equation (B23) gives

$$\sin\left(\theta_{m,0} + \frac{\pi}{4}\right) = \cos\left(\theta_{m,0} + \frac{\pi}{4}\right)(2\theta_{m,0} + \pi - 1) \quad (\text{B25})$$

or

$$\tan\left(\theta_{m,0} + \frac{\pi}{4}\right) = 2\theta_{m,0} + \pi - 1 \quad (\text{B26})$$

Solving equation (B26) by iteration and eliminating all solutions which are not in the first quadrant gives

$$\theta_{m,0} = 0.472 \quad (\text{B27})$$

Substituting equation (B27) into (B15) gives

$$\tau_{m,\max} \sin\left(0.472 - \frac{\pi}{4}\right) = \left|\frac{\tau_F}{R}\right| - \left|\frac{\tau_K}{R}\right| \quad (\text{B28})$$

or on multiplying by  $R$

$$\tau_{M,\max} \sin\left(0.472 - \frac{\pi}{4}\right) = |\tau_F| - |\tau_K| \quad (\text{B29})$$

or

$$0.308 \tau_{M,\max} = |\tau_K| - |\tau_F| \quad (\text{B30})$$

When equation (B30) is satisfied, the kinetic energy of the system becomes zero at  $\theta_{m,f}$ . For this condition a failure takes place because the rotor responds to a step command by moving from  $\theta_{m,0}$  to  $\theta_{m,f}$ . If  $\tau_{M,\max}$  is less than the value given by equation (B30), the rotor will move beyond  $\theta_{m,f}$  and a stepping failure will also occur. Therefore, to avoid a stepping failure the following must be observed for stepping clockwise with the scram spring:

$$0.308 \tau_{M,\max} > |\tau_K| - |\tau_F| \quad (\text{B11})$$

## APPENDIX C

### ACTUATOR SYSTEM EQUATIONS WITH BACKLASH

From Newton's second law the drum dynamics can be written

$$I_{DR} \ddot{\theta}_{DR} = \tau_L + \tau_F + \tau_K \quad (C1)$$

where

$$\tau_F = \tau_C + \tau_S \quad (C2)$$

Similarly, for the motor dynamics

$$I_m \ddot{\theta}_m + D_m \dot{\theta}_m = \tau_m + \tau_l \quad (C3)$$

$\tau_l$  is related to  $\tau_L$  as follows:

$$\tau_L = -R\tau_l \quad (C4)$$

Also

$$\left. \begin{aligned} \theta_M &= \frac{\theta_m}{R} \\ \dot{\theta}_M &= \frac{\dot{\theta}_m}{R} \\ \ddot{\theta}_M &= \frac{\ddot{\theta}_m}{R} \end{aligned} \right\} \quad (C5)$$

Multiplying equation (C3) by  $R$  and substituting equations (C4) and (C5) gives

$$R^2 I_m \ddot{\theta}_M + R^2 D_m \dot{\theta}_M = R\tau_m - \tau_L \quad (C6)$$



Define

$$I_M = R^2 I_m \quad (C7)$$

$$D_M = R^2 D_m \quad (C8)$$

$$\tau_M = R \tau_m \quad (C9)$$

The equation for the motor dynamics can then be written in the following form:

$$I_M \ddot{\theta}_M + D_M \dot{\theta}_M = \tau_M - \tau_L \quad (C10)$$

The backlash in this study is located in the coupling between the speed reducer and the drum. When the rotor is in the center of the backlash region,  $\theta_M = \theta_{DR}$ . For counterclockwise movement, backlash is taken up at the instant when both

$$\theta_M - \theta_{DR} - \frac{\delta}{2} = 0 \quad (C11)$$

and

$$\dot{\theta}_M - \dot{\theta}_{DR} \geq 0 \quad (C12)$$

For clockwise movement, backlash is taken up at the instant when both

$$\theta_M - \theta_{DR} + \frac{\delta}{2} = 0 \quad (C13)$$

and

$$\dot{\theta}_M - \dot{\theta}_{DR} \leq 0 \quad (C14)$$

At the instant when backlash is taken up, a torque  $\tau_L$  is transmitted between the load and the motor and its value must be such that  $\dot{\theta}_M = \dot{\theta}_{DR}$ . This value of  $\tau_L$  is approximated by the following equation:

$$\tau_L \cong K(\dot{\theta}_M - \dot{\theta}_{DR}) \quad (C15)$$

where  $K$  is a constant. The approximation becomes exact when  $K$  approaches infinity (hard collision between the backlash-causing members of the coupling mechanism). In this analysis, a value of  $180/\pi$  is used for  $K$ . Because of the approximation of equation (C15),  $\dot{\theta}_M$  is only approximately equal to  $\dot{\theta}_{DR}$  when backlash is taken up. But the approximation proved adequate for this analysis.

When backlash is traversed, both drum and motor move independently and then

$$\tau_L = 0 \quad (C16)$$

The frictional torques were calculated as follows:

For coulomb friction

$$\tau_C = -f_C r M g \frac{\dot{\theta}_{DR}}{|\dot{\theta}_{DR}|} \quad (C17)$$

The static friction when  $\dot{\theta}_{DR} = 0$  is

$$\tau_S = -(\tau_L + \tau_K) \quad (C18)$$

for  $|\tau_S| < \tau_{S, \max}$ . Otherwise ( $|\tau_S| \not< \tau_{S, \max}$ )

$$\tau_S = -\left(\frac{\tau_L + \tau_K}{|\tau_L + \tau_K|}\right) \tau_{S, \max} \quad (C19)$$

Where  $\dot{\theta}_{DR} \neq 0$ ,

$$\tau_S = 0 \quad (C20)$$

The maximum static friction is given by

$$\tau_{S, \max} = f_S r M g \quad (C21)$$

The spring torque  $\tau_K$  is assumed to be constant. For the case of a decoupled spring it has no effect on the drum and  $\tau_K$  is therefore zero.

The torque equations for a four-pole permanent magnet motor are derived in reference 4. They are listed below for each step command. The form containing both a

sine and cosine term was used in the analog study because it was found convenient for simulating stepping.

Step 0:

$$\tau_m = -\tau_{m, \max} \sin\left(\theta_m - \frac{\pi}{4}\right) = \frac{\tau_{m, \max}}{\sqrt{2}} (\cos \theta_m - \sin \theta_m) \quad (C22)$$

Step 1:

$$\tau_m = -\tau_{m, \max} \sin\left(\theta_m - \frac{3\pi}{4}\right) = \frac{\tau_{m, \max}}{\sqrt{2}} (\cos \theta_m + \sin \theta_m) \quad (C23)$$

Step 2:

$$\tau_m = -\tau_{m, \max} \sin\left(\theta_m - \frac{5\pi}{4}\right) = \frac{\tau_{m, \max}}{\sqrt{2}} (-\cos \theta_m + \sin \theta_m) \quad (C24)$$

Step 3:

$$\tau_m = -\tau_{m, \max} \sin\left(\theta_m - \frac{7\pi}{4}\right) = \frac{\tau_{m, \max}}{\sqrt{2}} (-\cos \theta_m - \sin \theta_m) \quad (C25)$$

For continued counterclockwise stepping, the procedure is repeated starting with step 0.

## APPENDIX D

### ANALOG DIAGRAM OF ACTUATOR SYSTEM

The analog diagram of the actuator system simulation is presented in figure 16 and described in this appendix.

The symbols used in figure 16 are defined as follows:  $e$  is a step command,

$\delta' = \frac{180}{\pi} \delta$	$\tau'_S = \frac{\tau_S}{400}$	$P11 = 0.4$
$\theta'_{DR} = \frac{180}{\pi} \left( \frac{\theta_{DR}}{10} \right)$	$P1 = \left( \frac{180 \times 400}{\pi \times 10^5} \right) \frac{1}{I_{DR}}$	$P12 = (10^{-2} \text{ rad/sec})^2$
$\dot{\theta}'_{DR} = \frac{180}{\pi} \left( \frac{\dot{\theta}_{DR}}{10^3} \right)$	$P2 = \frac{10^{-2} D_{DR}}{I_{DR}}$	$= (0.0157 \text{ steps/sec})^2$
$\theta'_m = \frac{180}{\pi} \left( \frac{\theta_m}{200} \right)$	$P3 = \left( \frac{180 \times 400}{\pi \times 10^5} \right) \frac{1}{I_M}$	$P13 = \tau'_C$
$\theta'_M = \frac{180}{\pi} \left( \frac{\theta_M}{10} \right)$	$P4 = \left( \frac{180 \times 400}{\pi \times 10^5} \right) \frac{1}{I_M}$	$P14 = \frac{10^3}{2 \times 10^6} R$
$\dot{\theta}'_M = \frac{180}{\pi} \left( \frac{\dot{\theta}_M}{10^3} \right)$	$P5 = 10^{-2} \frac{D_M}{I_M}$	$P15 = 0.707$
$\tau'_C = \frac{\tau_C}{400}$	$P7 = \frac{\delta'}{2}$	$P16 = 0.707$
$\tau'_L = \frac{\tau_L}{400}$	$P8 = \tau'_K$	$P17 = IC \text{ for } \theta'_m$
$\tau'_K = \frac{\tau_K}{400}$	$P9 = \tau'_S$	$P18 = \frac{\tau_{M, \max}}{400}$
$\tau'_M = \frac{\tau_M}{400}$	$P10 = 0.1$	$P19 = S$

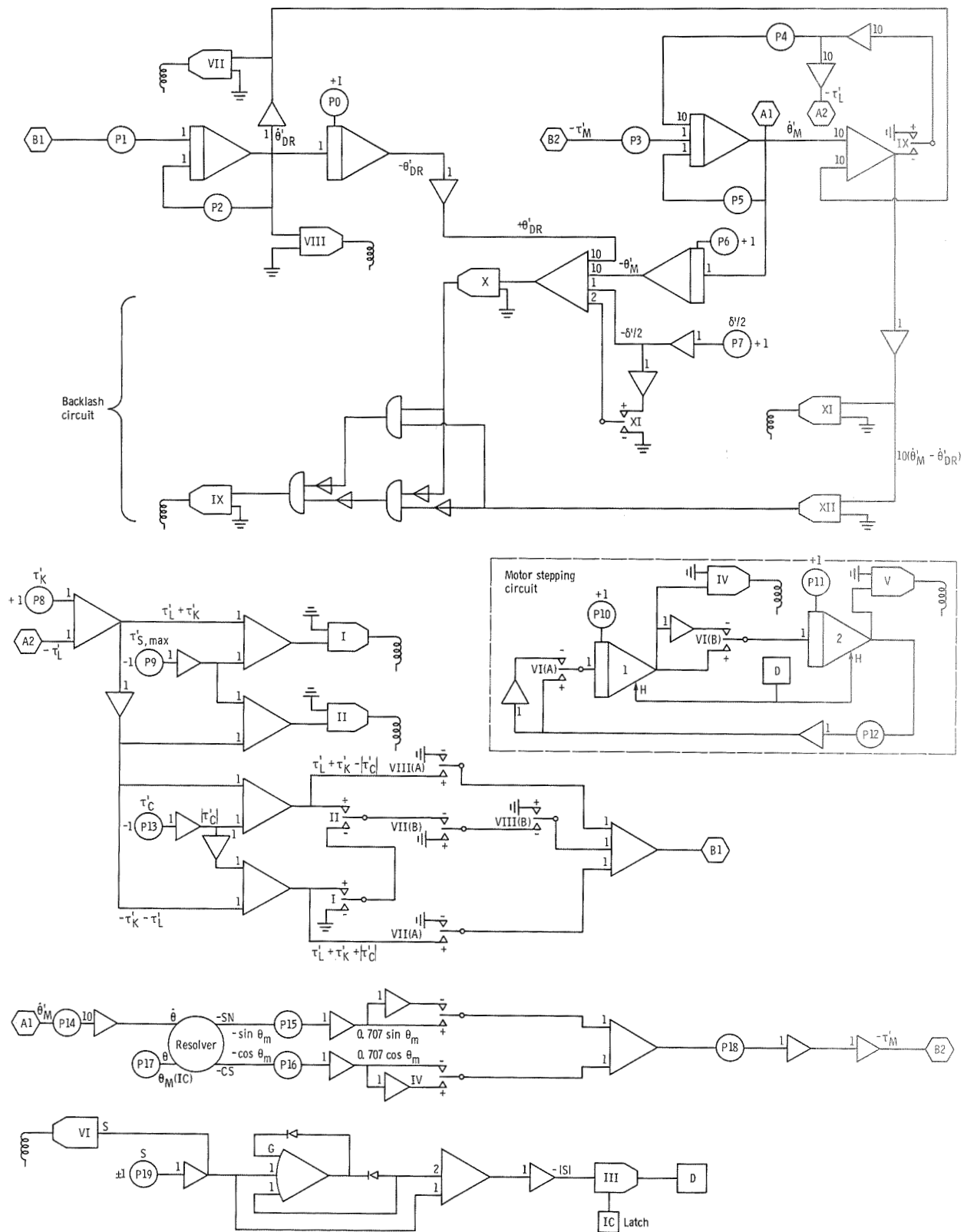


Figure 16. - Analog diagram of actuator system. Computer time is 100 times real time.

Roman numerals designate comparators and relays. The Roman numeral of each relay corresponds to the numeral of the comparator that drives the relay.

A positive input to a comparator drives it to state 1 and a negative input to state 0. At state 1 the comparator keeps its assigned relay switched to the plus terminal. State 0 switches the relay to the minus terminal.

Comparators I to XII perform the following functions: Comparators I and II test for  $|\tau_L + \tau_K| > \tau_{S, \max}$ : comparator I tests for  $-(\tau_L + \tau_K) > \tau_{S, \max}$ , comparator II tests for  $(\tau_L + \tau_K) > \tau_{S, \max}$ . Comparator III initiates stepping by placing integrators 1 and 2 into Operate whenever its input  $-|S|$  is not zero. The output of integrators 1 and 2 determines the state of comparators IV and V. Comparators IV and V in conjunction with their assigned relays perform the stepping. Comparator VI determines the stepping direction. When its input  $S$  is positive, stepping is in the counterclockwise direction. The direction is clockwise when  $S$  is negative. Comparators VII and VIII determine the direction of the coulomb frictional torque by testing for the direction of drum rotation  $\dot{\theta}_{DR}$ . Comparator IX is in state 1 when the system operates in the backlash region, where the motor is decoupled from the drum. When backlash is taken up, comparator IX is in state 0. With backlash taken up for counterclockwise motion, comparators X, XI, and XII are in state 1. These three comparators are in state 0 for clockwise motion.

The letter D is used to designate the logic drive which puts integrators 1 and 2 into Hold Operation when  $-|S| = 0$  or when computer is in Initial Condition.

Figure 16 gives a computer solution which is 100 times slower than the actual solution.

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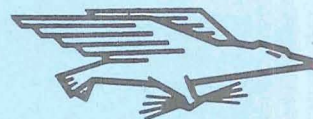
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